

# Short Term Hydrothermal Environmental/ Economic Dispatch Using Interior Point Method

*Despacho ambiental/económico de corto plazo utilizando  
el método de punto interior*

CARLOS ADRIÁN CORREA\*

RICARDO BOLAÑOS\*\*

ALEJANDRO GARCÉS\*\*\*

## ABSTRACT

This paper presents a methodology for solving the hydrothermal dispatch problem when environmental impacts considered. The solution is obtained through a high order interior point model for optimization that considers power flow (DC Model) constraints, reservoir and generation limits, as well as emission reduction. The goal is to minimize both emissions and costs, which becomes a multiobjective problem considering that both objectives are in conflict. To face this fact, a weighted sum method is used in order to obtain a Pareto optimal set of solutions so they can be analyzed by a decision maker. A 6-bus test system with two hydroelectric plants and two thermal plants is used to show the obtained results.

**Keywords:** Short term hydrothermal dispatch, optimization, interior point method, emissions, multiobjective.

## RESUMEN

Este artículo presenta una metodología de solución al problema de despacho hidrotérmico cuando se considera el impacto ambiental. La solución se obtiene mediante un método de punto interior de alto orden que considera restricciones de flujo de potencia (modelo DC), embalse, límites de generación y reducción de emisiones. El objetivo es minimizar emisiones y costos, con lo cual se convierte en un problema multiobjetivo, al considerar que ambos objetivos están en conflicto. Para enfrentar este hecho, se usa un método de suma ponderada para obtener un conjunto de soluciones Pareto óptimas, las cuales las puede analizar un tomador de decisiones. Se usa un sistema de prueba de seis barras con dos plantas hidráulicas y dos térmicas para mostrar los resultados obtenidos.

**Palabras clave:** despacho hidrotérmico de corto plazo, optimización, método de punto interior, emisiones, multiobjetivo.

DATE OF RECEPTION: JANUARY 21ST, 2012 • DATE OF APPROVAL: MARCH 17, 2012

\* Assistant Professor, Electrical Engineering Program, La Salle University, Colombia. E-mail: carcorrea@unisalle.edu.co.

\*\* Operation Coordination Analyst, Centro Nacional de Despacho, XM – Filial de ISA. E-mail: rabolanos@xm.com.co.

\*\*\* Assistant Professor, Universidad Tecnológica de Pereira. E-mail: alejandro.g.ruiz@ntnu.no.

## Introduction

Increase in power demand has led to research on new energy sources, such as tidal, wind, solar and geothermal power generation, among others. However, there is still a high dependence on traditional technologies: thermal and hydroelectric plants. Power generation based on burning of fossil fuels (such as coal, oil and gas), represents 67.1 % of the total generated electricity around the world (International Energy Agency, 2011), and is the main contributor to global warming through Nitrogen Oxide (NO<sub>x</sub>), Carbon Oxide (CO<sub>x</sub>) and Sulphur Oxide (SO<sub>x</sub>) emissions.

Hydrothermal dispatch consists on the scheduling of generation plants to supply a predefined demand over a certain period of time. For short-term dispatch, this period of time is usually one day (with 24 intervals) and the optimization problem is solved by minimizing only the operating cost, which, at the end, is only related to thermal plants and the needed fuel to be burned and converted to power in gas or steam turbines. This problem considers volumes in reservoirs, power limits in both thermal and hydroelectric plants, their power limits, and can also consider power flow in the transmission lines (Wood et al., 1984).

This paper deals with two objectives to be minimized: operating costs and emissions. The latter is included considering that nowadays the cost is not the only important issue, but the environmental aspects and impacts of the power system are also significant. Obviously, a big effort is required from governments, researches, the industry and all society in general to face the environmental problems present in our world today.

The classical problem of minimizing only the operating cost has been solved by linear programming techniques (Gorestin, 1991), evolutionary algorithms (EAs) such as genetic algorithms bacterial foraging algorithms (Christoforos, 2004; Gil, 2003; Farhat, 2009). The EAs have been widely used in the last years to solve all kinds of engineering problems and their potential has been confirmed in numerous papers of specialized journals. EAs differ from classical techniques in the way transitions are made; the former uses natural evolutionary principles and a set of solutions, while the former usually considers a single solution and a deterministic transition rule (Deb, 2001).

The multiobjective environmental economic dispatch problem has been solved, as shown by Spea (2010) and King (2011), and aims to return a Pareto optimal set of solutions ranging from a generation schedule with the lowest price but the highest emission level and another solution with the opposite characteristics.

The multiobjective algorithms are also used in a large variety of disciplines due to the conflicting nature of most objectives to be achieved during the process of solving different engineering problems (Deb, 2001; Correa, 2008).

## Problem Formulation

The problem of environmental/economic dispatch focuses on minimizing the operating cost of thermal plants during a time period, in such a way that water resources are properly handled, and also taking into account the minimization of gaseous emissions. The problem solved in this paper, besides considering the common hydrothermal dispatch constraints, considers the maximum power flow in the transmission lines based on the DC power flow.

The objective function of the problem has one term related to the operating cost of thermal plants as follows:

$$\sum_{t=1}^T \sum_{i=1}^{N_T} \psi_{(i,t)} (P_{(i,t)}) = \sum_{t=1}^T \sum_{i=1}^{N_T} \frac{a_{(i)}}{2} \cdot P_{(i,t)}^2 + b_{(i)} \cdot P_{(i,t)} \quad (1)$$

Where,

$a_{(i)}, b_{(i)}$ : Cost coefficients associated to plant  $i$ .

$P_{(i,t)}$ : Power generated in plant  $i$ , in the period of time  $t$ .

$T$ : Number of time periods.

$N_T$ : Number of thermal plants.

This objective function by itself represents only the economic part of the dispatch problem and is the one considered by the market operator in Colombia and most countries.

The emissions are considered in the problem through the following expression:

$$\sum_{t=1}^T \sum_{i=1}^{N_T} E_{(i,t)}(P_{(i,t)}) \quad (2)$$

With

$$E_{(i,t)}(P_{(i,t)}) = \alpha_t P_{(i,t)}^2 + \beta_t P_{(i,t)}^2 + \gamma_t \quad (3)$$

Where

$E_{(i,t)}(P_{(i,t)})$  Emissions function.

$\alpha_t, \beta_t$  and  $\gamma_t$  Emissions coefficients of thermal plant  $i$ .

The multiobjective problem has to minimize both objectives and when all the constraints are added, the problem formulation is the following:

$$\text{Min} \left[ \sum_{t=1}^T \sum_{i=1}^{N_T} \Psi_{(i,t)}(P_{(i,t)}), \sum_{t=1}^T \sum_{i=1}^{N_T} E_{(i,t)}(P_{(i,t)}) \right] \quad (4)$$

s.a.

$$V_{(j,t+1)} = V_{(j,t)} + \tau \cdot (A_{(j,t)} - Q_{(j,t)} - S_{(j,t)}) \quad (5)$$

$$P_{(j,t)} = \rho_{(j)} \cdot Q_{(j,t)} \quad (6)$$

$$P_{(i,t)} \Big|_{\text{nodo}(i)=k} + P_{(j,t)} \Big|_{\text{nodo}(j)=k} - D_{(k,t)} = \sum_{m=1}^N B_{km} \cdot \theta_m \quad (7)$$

$$-f_{km} \leq \frac{\theta_k - \theta_m}{x_{km}} \leq f_{km} \quad (8)$$

$$P_{\min(i,t)} \leq P_{(i,t)} \leq P_{\max(i,t)} \quad (9)$$

$$P_{\min(j,t)} \leq P_{(j,t)} \leq P_{\max(j,t)} \quad (10)$$

$$V_{\min(j,t)} \leq V_{(j,t)} \leq V_{\max(j,t)} \quad (11)$$

$$0 \leq S_{(j,t)} \leq S_{\max(j,t)} \quad (12)$$

Where:

- $V_{(j,t)}$  Water Volume in reservoir  $j$  during period  $t$ .
- $\tau$  Flow-volume conversion factor.
- $A_{(j,t)}$  Water inflow in reservoir  $j$  during period  $t$ .
- $Q_{(j,t)}$  Water flow through turbine in plant  $j$  during period  $t$ .
- $S_{(j,t)}$  Water spillage in plant  $j$  during period  $t$ .
- $\rho_{(j)}$  Flow-power conversion factor.
- $D_{(k,t)}$  Power Demand in bus  $k$ , during period  $t$ .
- $B_{km}$  Element  $k,m$  of susceptance matrix.
- $\theta_m$  Angle at bus  $m$ .
- $f_{km}, X_{km}$  Power flow and reactance between buses  $k$  and  $m$  respectively.

It should be noted that equations (8) to (11) limit power flows, thermal generation, hydroelectric generation, water volume in reservoirs and spillage, respectively.

To solve this multiobjective problem, both functions are taken into account using the weighted sum method, and thus, equation (4) takes the following form:

$$\text{Min } \lambda \sum_{t=1}^T \sum_{i=1}^{N_T} \Psi_{(i,t)} (P_{(i,t)}) + (1-\lambda) h \sum_{t=1}^T \sum_{i=1}^{N_T} E_{(i,t)} (P_{(i,t)}) \quad (13)$$

Where  $\lambda$  ranges from 1 to 0, and  $h$  is a conversion factor between volume (emissions) and monetary units.

This approach turns a multiobjective problem into a single objective optimization problem of easier solution.

The extreme values of  $\lambda$  represent the minimization of only one of the objectives. For example, if  $\lambda=1$ , the solved problem corresponds to the economic dispatch, and if  $\lambda=0$ , the minimization problem only takes into account the environmental considerations.

Equations (5) to (13) can be re-written in matrix notation:

$$\min [x]^T \cdot [A] \cdot [x] + [B]^T \cdot [x] + [C] \quad (14)$$

s.t.

$$[G] \cdot [x] - [b] = 0 \quad (15)$$

$$h' \leq [H] \cdot [x] \leq h'' \quad (16)$$

$$x^l \leq [I][x] \leq x^u \quad (17)$$

Which is a non-linear optimization problem that can be solved using interior point method, as shown in the following section.

## Interior Point Method

The Interior Point Method (IPM) starts from an initial point and calculates search directions towards a local optimum  $x^*$ . The IPM can be used for both linear and non-linear problems (Rider, 2004). Given that the hydrothermal problem is non-linear, the IPM for solving this type of problems will be presented.

### *Primal Dual Method*

The standard non-linear problem can be shown as follows:

$$\min f(x) \quad (18)$$

s.t.

$$g(x)=0 \tag{19}$$

$$h^l \leq h(x) \leq h^u \tag{20}$$

$$x^l \leq lx \leq x^u \tag{21}$$

For the analyzed case in this paper,  $f(x)$ ,  $g(x)$ ,  $h(x)$  and  $lx$ , are respectively: the objective function (13), constraints (5) to (7), constraint (8), and variable limits (9) to (11). Quantities  $nx$ ,  $ndx$ ,  $ndg$ ,  $ndh$  are defined as the number of variables, canalized variables, equality constraints and inequality constraints respectively. If slack variables are used ( $s_j > 0$ ) for converting inequality constraints into equality constraints, introducing logarithmic barrier terms into the objective function, and taking equality constraints into the objective function by means of the dual variables  $y$  and  $z_j$ , the resulting Lagrangean function is:

$$\begin{aligned} L_\mu = & f(x) - \mu^k \sum_{j=1}^{ndl} (\ln s_{1j} + \ln s_{2j}) - \mu^k \sum_{j=1}^{ndg} (\ln s_{3j} + \ln s_{4j}) - y^T (Ax - b) \\ & - z_1^T (-s_1 - s_2 - h^l + h^u) - z_2^T (-h(x) - s_2 + h^u) \\ & - z_3^T (-s_3 - s_4 - x^l + x^u) - z_4^T (-1x - s_4 + x^u) \end{aligned} \tag{22}$$

By applying Karush–Kuhn–Tucker’s first order optimality conditions  $F(w)$ , the following equations are obtained:

$$\begin{aligned} \nabla_{s_1} L_\mu &= -\mu^k e + S_1 z_1 & = 0 \\ \nabla_{s_2} L_\mu &= -\mu^k e + S_2 (z_1 + z_2) & = 0 \\ \nabla_{s_3} L_\mu &= -\mu^k e + S_3 z_3 & = 0 \\ \nabla_{s_4} L_\mu &= -\mu^k e + S_4 (z_3 + z_4) & = 0 \\ \nabla_{z_3} L_\mu &= S_3 + S_4 + x^l - x^u & = 0 \\ \nabla_{z_4} L_\mu &= 1x + S_4 - x^u & = 0 \\ \nabla_{z_1} L_\mu &= S_1 + S_2 + h^l - h^u & = 0 \\ \nabla_{z_2} L_\mu &= h(x) + S_2 - h^u & = 0 \\ \nabla_x L_\mu &= \nabla f(x) - J_g^T(x) y + J_h^T(x) z_2 + 1^T z_4 & = 0 \\ \nabla_x L_\mu &= -g(x) & = 0 \end{aligned} \tag{23}$$

And simplifying,  $F(w) = 0$ , then:

$$F(w) = \begin{bmatrix} -\mu^k e + S_1 z_1 \\ -\mu^k e + S_2 (z_1 + z_2) \\ -\mu^k e + S_3 z_3 \\ -\mu^k e + S_4 (z_3 + z_4) \\ S_3 + S_4 + x^1 - x^u \\ 1x + S_4 - x^u \\ S_1 + S_2 + h^1 - h^u \\ h(x) + S_2 - h^u \\ \nabla f(x) - J_g^T(x)y + J_h^T(x)z_2 + 1^T z_4 \\ -g(x) \end{bmatrix}, w = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ Z_3 \\ Z_4 \\ Z_1 \\ Z_2 \\ x \\ y \end{bmatrix} \quad (24)$$

For the hydrothermal problem described in equations (14) to (17):

$$\nabla f = 2A^1 x + B^{1T} \quad (25)$$

$$J_g = G, J_h = H \quad (26)$$

The *Jacobian* matrices associated to the constraints have constant values, a fact that simplifies the model. This system can be solved using Newton's method, as follows:

$$\Delta w^k = - \left[ J_F(w^k) \right]^{-1} F(w^k) \quad (27)$$

With the matrix  $J_F(w^k)$  as shown in (31). Diagonal matrices  $S_i$  y  $Z_i$  are formed by the values  $s_i$  and  $z_i$ .

$$\begin{bmatrix} Z_1 & 0 & 0 & 0 & 0 & 0 & S_1 & 0 & 0 & 0 \\ 0 & Z_1 + Z_2 & 0 & 0 & 0 & 0 & S_2 & S_2 & 0 & 0 \\ 0 & 0 & Z_3 & 0 & S_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_3 + Z_4 & S_4 & S_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 1 & 0 \\ I & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & J_h & 0 \\ 0 & 0 & 0 & 0 & 0 & 1^T & 0 & J_h^T & 2^T J_u & -J_g^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -J_g & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta s_1^k \\ \Delta s_2^k \\ \Delta s_3^k \\ \Delta s_4^k \\ \Delta z_3^k \\ \Delta z_4^k \\ \Delta z_1^k \\ \Delta z_2^k \\ \Delta x^k \\ \Delta y^k \end{bmatrix} = \begin{bmatrix} \mu^k e - S_1 z_1 \\ \mu^k e - S_2 (z_1 + z_2) \\ \mu^k e - S_3 z_3 \\ \mu^k e - S_4 (z_3 + z_4) \\ -S_3 - S_4 - x^1 + x^u \\ -1x - S_4 + x^u \\ -S_1 - S_2 - h^1 + h^u \\ -h(x) - S_2 + h^u \\ -\nabla f(x) + J_g^T(x)y - J_h^T(x)z_2 - 1^T z_4 \\ g(x) \end{bmatrix} \quad (28)$$

The term  $\nabla_x^2 L_\mu$ , is computed as:

$$\nabla_x^2 L_\mu = H_f(x^k) - \sum_{j=1}^{m_1} \gamma_j^k H_{g_j}(x^k) + \sum_{j=1}^{m_2} \lambda_{2,j}^k H_{h_j}(x^k) \quad (29)$$

For the hydrothermal problem considered, the term becomes  $\nabla_x^2 L_\mu = 2A'$ .

For initialization, variable update, barrier parameter reduction and convergence criteria see references Rider (2004) and Bolaños et al. (2007).

### ***High Order Interior Point Methods***

In general, high order methods predict the search direction, which is corrected afterwards to improve the method and accelerate the search process for the optimal point. In this paper, predictor corrector high order interior point method was used and its main ideas are described below.

#### ***Predictor Corrector Method (PCIPM)***

The PCIPM is a variation of the Dual-Primal method, improving the search directions to accelerate convergence. This method solves two linear systems in each iteration, using the same matrix shown in (28); the difference lays on the vector  $F(w^k)$  of this system. The two systems define the predictor and the corrector steps. If we add the second order terms to Newton's system:

$$J_F(w^k) \Delta w^k = -M(w^k) + \mu^k u - \Delta \quad (30)$$

Where:

$$\Delta = [\Delta S_1 \Delta S_1, \Delta S_2 (\Delta Z_1 + \Delta Z_2), \Delta S_3 \Delta S_3, \Delta S_4 (\Delta Z_3 + \Delta Z_4), 0, 0, 0, 0, 0, 0]^T \quad (31)$$

Three components for the search directions are obtained from (30). These directions are divided into the predictor and corrector steps.

$$\Delta w^k = \Delta w_{af}^k + \Delta w_{ce}^k + \Delta w_{co}^k$$

$$\left\{ \begin{array}{l} 1 \quad 4 \quad 2 \quad 4 \quad 3 \\ \text{Predictor} \quad \text{Corrector} \end{array} \right. \quad (32)$$

$\Delta w_{af}^k$  is the predictor direction or *affine – scaling* direction, where  $\mu^k = 0$ ,  $\Delta w_{ce}^k$  is the central direction choosing an appropriate  $\mu^k$  and  $\Delta w_{co}^k$  is the corrector direction.

### Predictor Step

The *affine–scaling* direction is calculated by solving (30), taking into account only the first term in the right side of the equation. This is used to approximate the non-linear terms  $\Delta$  and to estimate a value for the barrier parameter  $\mu^k$ , which are used in the corrector step. Sizes of the dual-primal step, in the *affine–scaling* direction,  $\alpha_p^{af}$  y  $\alpha_d^{af}$ , are calculated as shown in Bolaños et al (2007). The complementary gap of the predictor step is given by:

$$\begin{aligned} \rho^{af} = & (z_1 + \gamma \alpha_d^{af} \Delta z_1^{af})^T (s_1 + \gamma \alpha_p^{af} z_1^{af}) + \\ & (z_1 + z_2 + \gamma \alpha_d^{af} (\Delta z_1^{af} + \Delta z_2^{af}))^T (s_2 + \gamma \alpha_p^{af} \Delta s_2^{af}) + \\ & (z_3 + \gamma \alpha_d^{af} \Delta z_3^{af})^T (s_3 + \gamma \alpha_p^{af} \Delta s_3^{af}) + \\ & (z_3 + z_4 + \gamma \alpha_d^{af} (\Delta z_3^{af} + \Delta z_4^{af}))^T (s_4 + \gamma \alpha_p^{af} \Delta s_4^{af}) \end{aligned} \quad (33)$$

An estimate of  $\Delta^{af}$  is:

$$\mu^{af} = \min \left\{ \left[ \frac{\rho^{af}}{\rho^k} \right], 0, 2 \right\} \frac{\rho^{af}}{2(ndx + ndh)} \quad (34)$$

### Corrector Step

With the results of the predictive step, the direction  $\Delta w^k$  can be calculated, solving the complete system (30). The corrector step calculates simultaneously the directions  $\Delta w_{ce}^k$  and  $\Delta w_{co}^k$ . Finally, the additional effort in the PCIPM is present in the calculation of  $\Delta w_{af}^k$ ,  $\mu^{af}$ ,  $\alpha_p^{af}$  and  $\alpha_d^{af}$ ; however, the advantages lay on the reduction of the number of iterations, which leads to a total reduction in the computational time (Bolaños et al., 2007).

## Simulation and Results

The proposed PCIPM was implemented on Matlab® 2009, and used to solve the problem described in equations (5) to (13) with variations of  $\lambda$  from 0 to 1, to obtain a complete set of trade-off solutions. The methodology was tested on the 6 bus hydrothermal system in Wood et al (1984) and with the modifications described in Garcés (2006) and Bolaños et al (2007). The system has two hydroelectric plants, two thermal plants, 11 transmission lines, 3 load buses and 6 time intervals. For the sake of space, we refer the reader to Wood (1984), Garcés (2006) and Bolaños et al. (2007), where data for the power system, hydroelectrical and thermal plants are shown.

The first test is done by finding the minimum values of emissions ( $\lambda=0$ ) and cost ( $\lambda=1$ ). For these extreme values of  $\lambda$  the multiobjective problem is turned into a single objective problem and it is used to find the extreme points of the trade-off set of solutions. For the environmental dispatch problem ( $\lambda=0$ ), the obtained solution is 2984.4 (volume units) and the economic dispatch problem ( $\lambda=1$ ) leads to a cost of \$12,204, coinciding with the results in Garcés (2006) and Bolaños et al. (2007).

The cost and emissions values for a variation of  $\lambda$  in steps of 0.1 are shown in Table 1. Data shown is the total for the whole period of time.

Table 1. Trade-Off Set of Solutions

$\lambda$	Cost (\$)	EMISSIONS (VOL.)
0	12,381	2984.4
0.1	12,370	2985.0
0.2	12,358	2987.2
0.3	12,344	2991.8
0.4	12,328	3000.3
0.5	12,311	3015.1
0.6	12,290	3040.3
0.7	12,267	3083.7
0.8	12,242	3160.8
0.9	12,217	3305.4
1.0	12,204	3602.2

For variations of  $\lambda$  in steps of 0.01, although data is not displayed, Figure 1 shows the shape of the trade-off solutions.

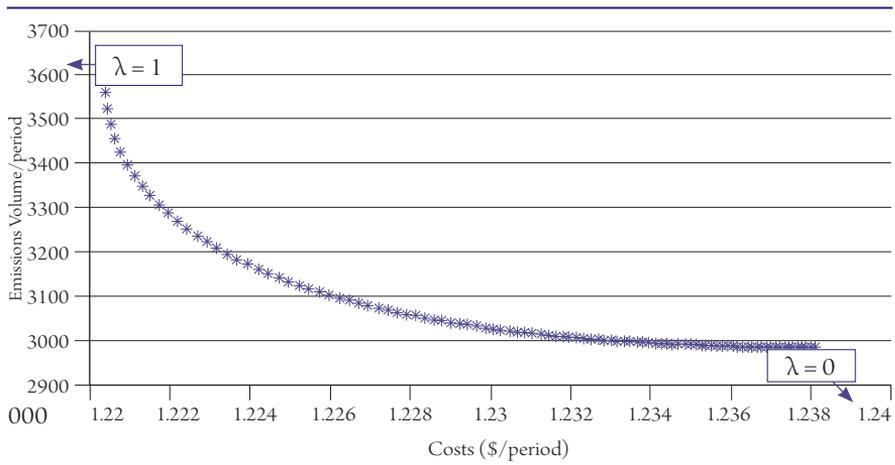


Figure 1. Emissions/Cost Pareto Front

It is important to note that the extreme points of the trade-off set of solutions (Pareto Optimal) correspond to the decoupled environmental and economic dispatch problem, and among them, there are multiple possible solutions with different costs and levels of emissions. Figure 1 also contains all solutions shown in Table 1.

## Conclusions

The proposed methodology allows obtaining not one single solution, but a complete set of Pareto Optimal solutions for the short term environmental/economic dispatch problem, which is an advantage for the system operator if different scenarios for emissions need to be considered. This approach allows a decision maker to select the appropriate generation schedule for thermal and hydroelectric power plants for different time periods, according to higher level information. If only economic considerations are included, higher emission levels are released into the atmosphere, increasing environmental problems and accelerating global warming, and that's why it is important for environmental constraints to start being analyzed and regulated.

The interior point method shows excellent convergence properties for solving the short term hydrothermal dispatch problem. This method returns an iterative

calculus based solution due to the use of the Newton method during the convergence process. Predictor Corrector high order method accelerates convergence when compared to the Dual-Primal Method.

Multiobjective problems have different approaches; in this case conflicting objectives (cost vs. emissions) are treated as a weighted sum to convert the formulation into a single objective problem. This is a simple but powerful alternative to face the problem. Other methods are based on evolutionary algorithms such as genetic algorithms; particle swarm and their formulation can be adapted to deal with Multiobjective problems (NSGA-II, MOGA, SPEA, DPGA, etc.)

## References

- Bolaños, R.; Correa, C. and Garces, A. (2007). *Estudio comparativo de métodos de puntos interiores no lineales aplicados al problema del despacho hidrotérmico*. Simposio Internacional Sobre Calidad de Energía Eléctrica. Universidad Nacional de Colombia, Manizales, Colombia.
- Christoforos, E.; et al. (2004). A Genetic Algorithm Solution Approach to the Hydrothermal Coordination Problem. *IEEE Transactions on Power Systems*, 19 (2), 1356-1364.
- Correa, C. (2008). *Planeamiento multiobjetivo de la expansión de la transmisión considerando múltiples escenarios de generación*. Master's in Electric Engineering, Universidad Tecnológica de Pereira. [Master's Thesis].
- Deb, K. (2001). *Multiobjective Optimization Using Evolutionary Algorithms*. West Sussex, England: John Wiley & Sons.
- Farhat, I. and El-Hawary, M. (2009). *Short-term hydro-thermal scheduling using an improved bacterial foraging algorithm*. Electric Power and Energy Conference, IEEE. Montreal, Quebec, Canada.
- Garcés, A., Galvis, J. and Gómez, O. (2006). Algoritmo evolutivo diferencial aplicado al problema de despacho hidrotérmico. *Scientia et Technica*, 12(32), 187-192.
- Gil, E.; Bustos, J., and Rudnick H. (2003). Short-Term hydrothermal generation scheduling model using a genetic algorithm. *IEEE Transactions on Power Systems*, 18(4), 1256-1264.
- Gorenstin, B., Campodonico, N., Costa, J. and Pereira, M. (1991). *Stochastic Optimization of a Hydro-thermal System Including Network Constraints*. Power Industry Computer Application Conference. IEEE. Baltimore, MD, USA.
- International Energy Agency. (2011). *Key World Energy Statistics, 2011*.

- King, R., Rughooputh, H. and Deb, K. (2011). *Solving the Multiobjective Environmental / Economic Dispatch Problem with Prohibited Operating Zones Using NSGA-II*. Pacific Rim Conference on communications, computers and signal processing, Victoria, Canada.
- Rider, M. (2004). *Método de puntos interiores para optimización en sistemas eléctricos*. Seminario de optimización en sistemas de potencia. Pereira, Colombia.
- Spea, S., Abou, E., and Abido M. (2010). *Multiobjective Differential Evolution Algorithm for Environmental-Economic Power Dispatch Problem*. Energy Conference and Exhibition (EnergyCon), IEEE. Manama, Saudi Arabia.
- Wood, A. and Wollenberg, B. (1984). *Power Generation Operation and Control*. New York: John Wiley.